

TeachSpin is back from the APS March in-person meeting in Chicago, where we were delighted to return to face-to-face (mask-to-mask?) encounters with old friends and newcomers. And the vendor exhibit gave us the opportunity to show off our newest instrument,

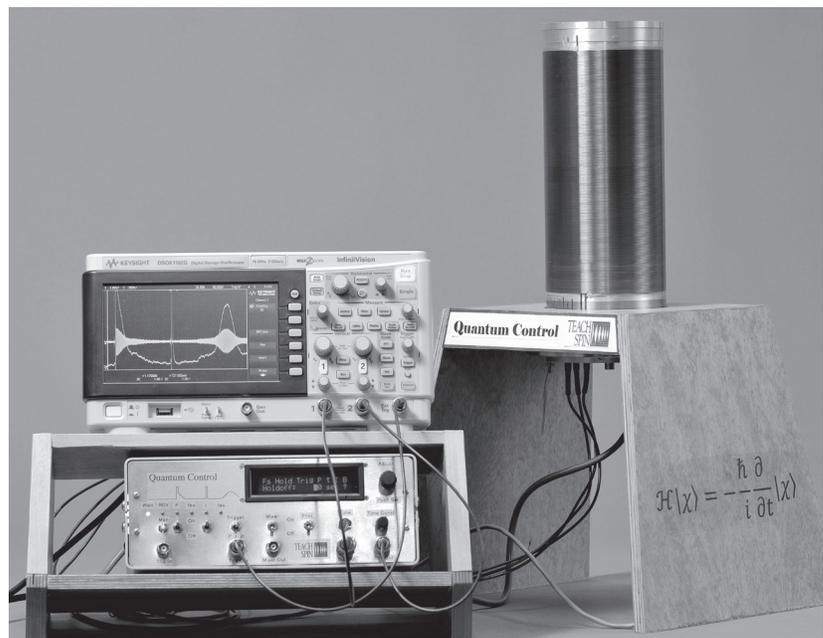
< Quantum | Control >

which is what we've named our attractively-priced tabletop apparatus for proton-spin quantum manipulation. We offer it as a contribution to laboratory education in the 'second quantum revolution' that's creating so much buzz in the physics community.

What's happening is that a generation of quantum capabilities is moving out of the lab, into new growth industries such as *quantum communication*, *quantum cryptography*, and *quantum computing*. In each case, unique post-classical features of quantum phenomena are being harnessed to create new possibilities of considerable technological importance.

As physicists, we want to make the point that the foundation on which any quantum computing is based has to be 'quantum hardware', some physical system evolving according to the laws of quantum physics. Various undergraduate experiments might serve to prepare students to participate in this revolution. We offer here a highly affordable quantum-hardware apparatus for hands-on experimentation. In our Quantum Control experiment, we focus on both the free, and the driven, *time-evolution of prepared quantum systems*. Contrary to some misconceptions among the public, we make the point that state-function evolution within quantum physics is *deterministic* in character.

In Quantum Control, we give students the chance to prepare, and then to manipulate, a two-level quantum system. It's not a N -qubit quantum computer, but it could fairly be called a 1-qubit *quantum register*. It can be prepared in one (or the other) of its two states, and (crucially) it can be put, controllably, into the *superposition state* that's so important to the whole of the second quantum revolution.



How it's done

For reasons of practicality, we've chosen to do this in an 'ensemble experiment', whose active ingredient is a whole collection of proton spins. The spin magnetic moments of these protons interact with a static and highly homogeneous magnetic field, with a magnitude of about 2000 μT (= 0.002 Tesla or 20 gauss), produced by a d.c.-excited and highly-corrected solenoid. Spin-up and spin-down protons form the 'basis states' in our two-level system, and they lie separated in energy by $\Delta E_{\text{mag}} \approx h \cdot (90 \text{ kHz})$ due to this magnetic field.

We initialize our quantum register by mere waiting – out of $\approx 4 \times 10^{24}$ protons (H-atom nuclei in our 58-cm³ liquid-water sample) we get, within 10 s, a Boltzmann-equilibrium population *difference* leaving $\approx 3 \times 10^{16}$ 'extra' protons in the lower-energy spin state, aligned along the field. These excess spins form our active sample.

In that environment, we are able to affect the spin-states' time evolution by adding fully-controlled *non-constant* magnetic fields, which can drive quantum transitions in our two-level system. Such transitions occur with highest probability when the frequency f of those fields is chosen to

match the Bohr criterion $h \cdot f = \Delta E_{\text{mag}}$. But contrary to impressions left by some textbooks, such driven transitions are not an all-or-nothing proposition; in fact, the ones most useful here are deliberately crafted to turn the initial state of spins-along-field into a *quantum superposition* of spin-up and spin-down states. Students will see why, and how, this is accomplished with a ' $\pi/2$ pulse', and what's so ' $\pi/2$ ' about it.

Not every student knows that the superposition state of spin-along-z and spin-opposite-to-z can be an eigenstate of spin-along-x. Better still, a superposition of spin states of two distinct energies yields a state with observable properties, such as $\langle S_x \rangle$, that are non-zero and also time-varying. In fact, our spin-superpositions create a time-varying expectation value of the sample's magnetization, of sufficient size to produce a directly-detectable electronic signal.

Our apparatus lets students see quantum physics happening on a *timeline*. We focus on *preparation, intervention, and readout* phases, separated in time by hundreds of milliseconds. Apart from brief (< 1 ms) external perturbations, the proton spin system evolves freely. Our apparatus includes an electronic controller which allows full control, along that timeline of state evolution, of the *time-location, time-duration, frequency, and amplitude*, for both the preparation and the intervention phases.



Typical results

Our apparatus, comprising solenoid, sample with pre-amp, plus electronic controller, requires only line power and an oscilloscope to give easily detectable signals. A few minutes' initial work will determine the solenoid current needed for putting proton spin-precession signals at the frequency of the detection system's peak sensitivity. A typical timeline calls for a preparation phase consisting of 6-10 s of thermalization, plus a crafted preparation-pulse yielding the desired superposition-state. Results from the time-evolution of this superposition are immediately detectable (giving the signals seen at the left of the 'scope trace below). But the heart of the experiment is another phase we call *intervention*, the effort to drive a quantum transition affecting the superposition state (this occurs in the middle of the figure below). The success of driving that quantum transition is directly visible as a resuscitated signal (visible at the right side of the figure, in what's called a 'spin echo' in applications to nuclear magnetic resonance). Of course there's a neat bit of two-level theory which accounts for the details of these signals, and which shows that the strength of these recovered signals is a *direct measure of the transition probability* caused by the intervention phase of the experiment. That probability is maximized by an 'on-resonance π -pulse', and students will learn just what that means.

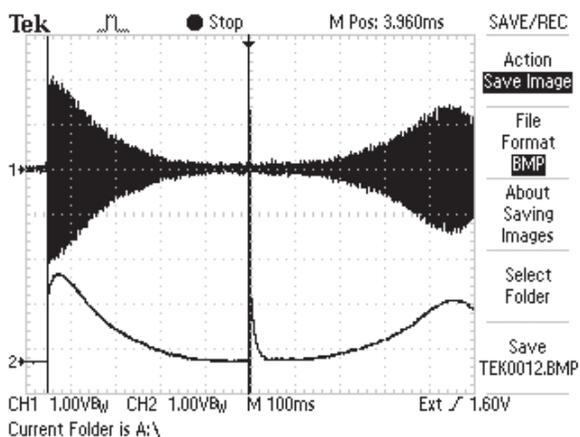


Fig. 1: Output signal (above) and its envelope (below) for a single 'run' of the Quantum-Control experiment. In this case, the state preparation ends at left; 450 ms later comes the intervention (at center); the resuscitated signal appears at 450 ms later still, at right.

The raw 'observable' in this experiment is the occurrence of an oscillatory signal near 90 kHz, visible with very good signal-to-noise ratio, and whose individual cycles are in *one-to-one correspondence* with the turns-of-precession of protons in the sample. An expansion of the time axis (by a factor of 4000!) shows these cycles of the signal: at the center of the spin-echo, signal amplitudes near 2 V lie atop a noise floor of rms measure <40 mV, for a 50:1 signal-to-noise ratio on a single 'shot'.

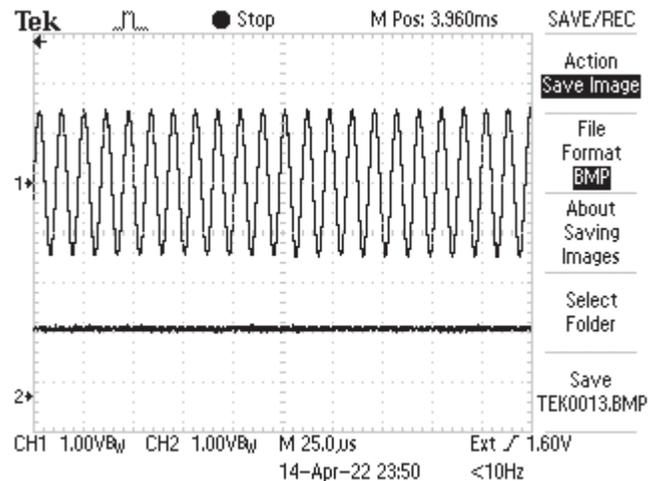


Fig. 2: As above, but centered on the time of the 'echo' signal, and horizontally expanded by a factor of 4000. The precession cycles of the resuscitated signal are individually visible.

Our controller also makes available an 'envelope' signal giving the local-average amplitude of these oscillations. The peak value of that envelope gives the 'echo strength', which serves as the dependent variable for a whole class of experiments. In such experiments, the *independent* variables include the time-location, and the duration, of the intervention, as well as the amplitude and the frequency of the intervention's oscillatory magnetic field. Below are some plots showing a few of these dependencies.

If we vary only the **amplitude** of the oscillating intervention, while holding its duration and frequency fixed, we get a strength-of-echo which (initially) grows quadratically with the amplitude of the intervention. That growth is just as predicted from lowest-order perturbation theory. But we can easily explore the

regime where perturbation theory's predictions *fail*, and where in fact the signal strength starts to *drop* – in fact, we can drive the signal all the way back to zero! Better still, in a highly-justifiable rotating-wave approximation, *we can solve the problem non-perturbatively*, and that gives the solid curve in the plot below. Each data-point in the plot is the result of a single ‘run’ of the experiment, and we can do a fresh run every 6 or 10 seconds.

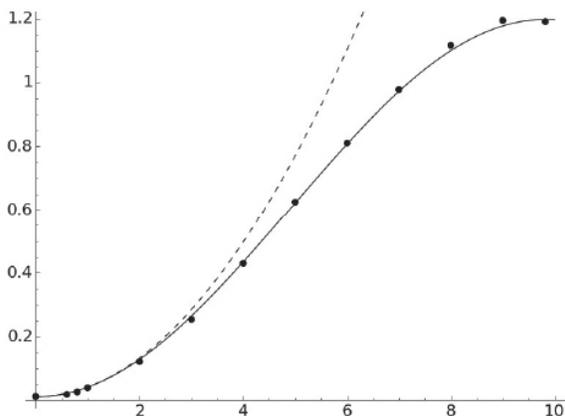


Fig. 3: Plotting the echo strength (vertically, in V) as a function of the amplitude (in V) of the drive for the intervention pulse. The dashed curve gives the prediction of first-order perturbation theory, while the solid curve gives the non-perturbative prediction.

The continuous curve in the model above has a vertical scale empirically matched to the signal strength we see, but the horizontal scale is *not* adjusted. Rather, because the oscillating magnetic field strength is known in actual microtesla (μT), quantum-mechanical theory makes a firm prediction from *first principles* of what it should take to reach the first maximum, and the subsequent minimum, of the curve shown. Theory and observation match at the $\approx 1\%$ level. Similar agreement can be had when the amplitude of the intervention is held fixed, but its **duration** is varied instead.

Next, we can fix the amplitude and duration of the intervention to put us at that ‘first-maximum’ location for on-resonance excitation, but then we can vary the **frequency** of the intervention waveform at will. Now plotting the strength-of-echo as dependent variable, we get a ‘Rabi lineshape’, showing the probability of driving a quantum transition:

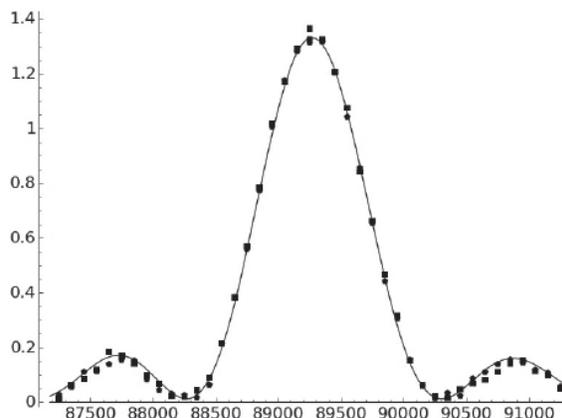


Fig. 4: Plotting the echo strength (vertically, in V) as a function of the frequency (in Hz) of the intervention pulse. The solid curve is the first-principles prediction of non-perturbative quantum mechanics, with nothing adjusted but an overall vertical scaling.

Again, each plotted point comes from a single ‘run’ of the experiment. Here too we overlay a theoretical prediction that comes straight from quantum mechanics; again, we only adjust the vertical scale of our prediction. Everything else about the predicted curve – including its shape, its width, even the location and height of its sidelobes – comes from the theory, *without any fitting required*. Clearly, students will learn that quantum physics allows the predictable control of quantum systems.

The plot above is obtained using a ‘ π -pulse’ to maximize the transition probability at line center, meaning the product of a certain intervention-strength and its time-duration has been adjusted to be π radians. But relative to the numbers used above, it’s easy to halve the amplitude, and double the duration,

of the intervention. Theory predicts that this new combination also gives a π -pulse, and hence again gives a maximal signal at line center. But now a frequency scan will reveal a different panorama, one having the same shape as above, but with only *half the width* in frequency space:

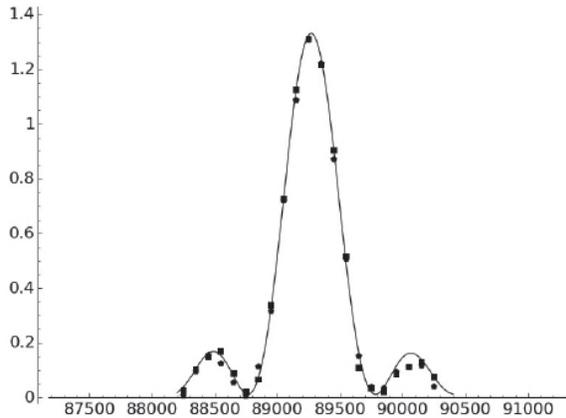


Fig. 5: Echo strength (vertically) as a function of the frequency (horizontally), exactly as in Fig. 4 -- except that here, the intervention pulse has half the amplitude, and double the duration, compared to the previous case. Again, the solid curve is the first-principles prediction of non-perturbative quantum mechanics.

Students might recognize this outcome as a concrete illustration of what is sometimes called the ‘energy-time uncertainty principle’, and they’ll certainly learn the spectroscopic lesson that higher resolution-in-frequency requires the use of a longer interaction-in-time.

Finally, a well-known obstacle to almost any form of ‘applied quantum operations’ is the process of ‘decoherence’, the process by which a quantum system in effect loses its memory. This too can be illustrated in our Quantum Control apparatus. We can optimize the preparation and intervention we exercise on our

proton sample, but then vary the time interval τ between the two. In each case, we get our resuscitated signal not right after the intervention, but at a time τ later than that, at net time 2τ after the preparation. We find our signal strength drops, approximately exponentially, as a function of that net evolution time:

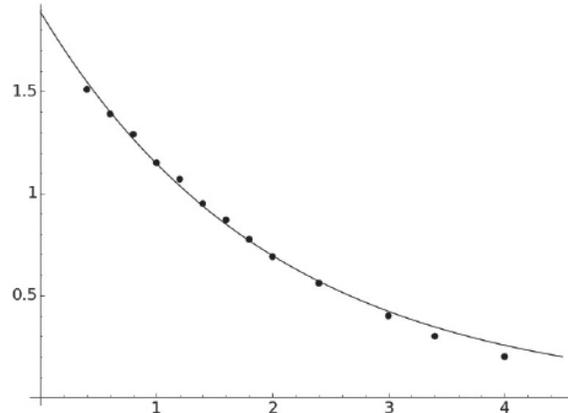


Fig. 6: Plotting the echo strength (vertically, in V) as a function of the time delay 2τ (in seconds) from preparation to ‘read-out’ or observation of the echo. The solid curve is an exponential model, with time-constant 2.0 s, describing the short-term behavior.

If we overlay a single exponential on this data, we get a decoherence time-constant of about 2.0 s for our sample. That time is impressively long, all the more so relative to the sub-millisecond duration of the preparation and intervention pulses we apply.

There’s much more that can be varied, investigated, and measured using our Quantum Control apparatus. We expect to have units available, in quantity, by late summer 2022, and we are offering the first units at an introductory price of \$2995. Let us know what questions you have about our newest, spin-teaching, TeachSpin tool.



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